The triple integral over a box $B = [a, b] \times [c, d] \times [p, q]$ is given by

$$\int \int \int_B f(x, y, z) \, dV.$$

This may be evaluated in any order.

A simple region $W$ in $\mathbb{R}^3$ is a region consisting of of the points between the surfaces $z = z_1(x, y)$ and $z = z_2(x, y)$ (where $z_1(x, y) \leq z_2(x, y)$) lying over a domain $D$ in the $xy$-plane. The triple integral over $W$ is the iterated integral

$$\int \int \int_W f(x, y, z) \, dV.$$

The average value of a function $f(x, y, z)$ on a simple region $W$ of volume $V$ is

$$\bar{f} = \frac{\int \int \int_W f(x, y, z) \, dV}{V},$$

where $V =$

**Problem Set**

1. Evaluate the triple integral of $f(x, y, z) = x(y + z)^2$ over the box $B = [0, 2] \times [2, 4] \times [-1, 1]$.

2. Integrate $f(x, y, z) = z$ over the region lying below the upper hemisphere of radius 4 and above the triangle in the $xy$-plane bounded by the lines $x = 1$, $y = 0$ and $x = y$.

3. Find the volume of the solid in the octant $x, y, z \geq 0$ bounded by the planes $x + y + z = 1$ and $x + y + 2z = 1$.

4. Find the average of $f(x, y, z) = xy \sin(\pi z)$ over the cube $0 \leq x, y, z \leq 1$. 
The triple integral over a box $B = [a, b] \times [c, d] \times [p, q]$ is given by

$$\int\int\int_B f(x, y, z) \, dV = \int_a^b \int_c^d \int_p^q f(x, y, z) \, dz \, dy \, dx.$$ 

This may be evaluated in any order.

A simple region $W$ in $\mathbb{R}^3$ is a region consisting of the points between the surfaces $z = z_1(x, y)$ and $z = z_2(x, y)$ (where $z_1(x, y) \leq z_2(x, y)$) lying over a domain $D$ in the $xy$-plane. The triple integral over $W$ is the iterated integral

$$\int\int\int_W f(x, y, z) \, dV = \int\int_D \left( \int_{z_1(x, y)}^{z_2(x, y)} f(x, y, z) \, dz \right) \, dA.$$ 

The average value of a function $f(x, y, z)$ on a simple region $W$ of volume $V$ is

$$\mathcal{F} = \frac{1}{V} \int\int\int_W f(x, y, z) \, dV$$

where $V = \int\int\int_W 1 \, dV$. 

\[ \]
1. Evaluate the triple integral of \( f(x, y, z) = \frac{x}{(y+z)^2} \) over the box \( B = [0, 2] \times [2, 4] \times [-1, 1] \).  
Answer: \(-2 \ln 5 + 4 \ln 3\)

2. Integrate \( f(x, y, z) = z \) over the region lying below the upper hemisphere of radius 4 and above the triangle in the \( xy \)-plane bounded by the lines \( x = 1, y = 0 \) and \( x = y \).  
Answer: \(\frac{25}{12}\)

3. Find the volume of the solid in the octant \( x, y, z \geq 0 \) bounded by the planes \( x + y + z = 1 \) and \( x + y + 2z = 1 \).  
Answer: \(\frac{1}{12}\)

4. Find the average of \( f(x, y, z) = xy \sin(\pi z) \) over the cube \( 0 \leq x, y, z \leq 1 \).  
Answer: \(\frac{1}{2\pi}\)